

التمرين: 01

ليكن غاز يخضع لحلقة الديزل حيث

$$P_1 = 1 \text{ Bar} = 10^5 \text{ Pa}, T_1 = 300 \text{ K}$$

$$V_1 / V_2 = 14$$

$$T_3 = 1340 \text{ K}$$

$$T_4 = 556 \text{ K}$$

$$R = 8,31 \text{ J.mol}^{-1}.\text{K}^{-1}, \gamma = C_p/C_v = 1,4, C_v = 20,8 \text{ J.mol}^{-1}.\text{K}^{-1}.$$

1- أحسب T_2

2- أحسب كمية الحرارة لكل تحول

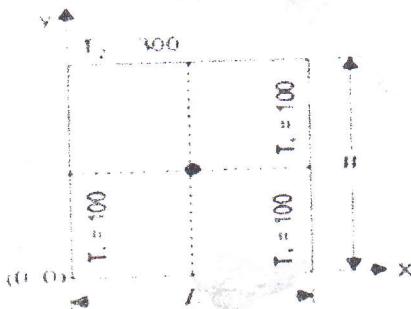
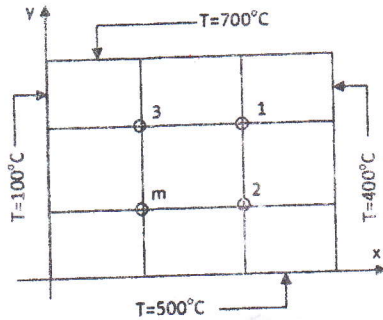
3- أحسب عمل الحلقة والمردود

التمرين: 02

1) باستعمال طريقة المتفرقات المنتهية

(Méthode des différences finies)

أحسب T_1, T_2, T_3 حيث $346^\circ\text{C} = T_m$ $y\Delta = x\Delta$



2) ليكن جدار على شكل مستطيل أبعاده $L=1\text{m}$ و $W=2\text{m}$

بالاستعانة بطريقة فصل المتغيرات وتحويل فورييه

Méthode de Séparation des variables et transform de Fourier

أحسب $T(0.5,1)$

نكتفي ب 4 نقاط غير معدومة

$$H \equiv \frac{T - T_1}{T_2 - T_1}$$

المدرسة العليا لأساتذة التعليم التكنولوجي - سكيكدة-
الاختبار الإستدراكي في الطاقة
 سنة رابعة PEST

التمرين: 01

$$1-2 \text{ est adiabatique} \Rightarrow T_1 \cdot V_1^{\gamma-1} = T_2 \cdot V_2^{\gamma-1} \Rightarrow T_2 = T_1 \cdot \left(\frac{V_1}{V_2} \right)^{\gamma-1} \approx 300 \times 14^{1.4-1} \approx 862 \text{ K}$$

$$Q_{12} = 0 \text{ car } 1-2 \text{ est adiabatique}$$

$$Q_{23} = n \cdot C_p \cdot (T_3 - T_2) \text{ car } 2-3 \text{ est isobare} \Rightarrow \text{pour } n = 1 \text{ mole on a } Q_{23} = \gamma \cdot C_v \cdot (T_3 - T_2) \approx 1.4 \times 20.8 \times (1340 - 862) \approx 14 \text{ kJ}$$

$$Q_{34} = 0 \text{ car } 3-4 \text{ est adiabatique}$$

$$Q_{41} = C_v \cdot (T_1 - T_4) \text{ car } 4-1 \text{ est isochore} \Rightarrow Q_{41} \approx 20.8 \times (300 - 556) \approx -5 \text{ kJ}$$

$$\Delta U_{\text{cycle}} = 0 \text{ pour tout cycle}$$

$$1^{\text{er}} \text{ principe : } W + Q = \Delta U_{\text{cycle}} = 0$$

$$W = -Q \Leftrightarrow W = -(Q_{23} + Q_{41}) \approx -(14 \cdot 10^3 - 5 \cdot 10^3) \approx -8.6 \text{ kJ} : \text{ le travail est négatif, preuve qu'il s'agit bien d'un cycle moteur !}$$

$$\eta \triangleq \left| \frac{\text{travail fourni}}{\text{chaleur absorbée}} \right| \Leftrightarrow \eta = \left| \frac{W}{Q_{23}} \right| \approx 8.6 \cdot 10^3 / 14 \cdot 10^3 \approx 0.62$$

التمرين: 02

$$\theta(x, y) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} + 1}{n} \sin \frac{n\pi x}{L} \frac{\sinh(n\pi y/L)}{\sinh(n\pi W/L)}$$

$$\frac{T(0.5, 1) - 100}{300 - 100} = 0.0549 \rightarrow T(0.5, 1) = 111.5$$

$$\begin{aligned} n=1 &\rightarrow 2 \sin\left(\frac{\pi x}{L}\right) \cdot \frac{\sinh\left(\frac{\pi y}{L}\right)}{\sinh\left(\frac{\pi W}{L}\right)} = 2 \sin\left(\frac{\pi}{2}\right) \cdot \frac{\sinh\left(\frac{\pi}{2}\right)}{\sinh\left(\frac{\pi}{1}\right)} = 0.862 \\ n=3 &\rightarrow \frac{2}{3} \sin\left(\frac{3\pi x}{L}\right) \cdot \frac{\sinh\left(\frac{3\pi y}{L}\right)}{\sinh\left(\frac{3\pi W}{L}\right)} = \frac{2}{3} \sin\left(\frac{3\pi}{2}\right) \cdot \frac{\sinh\left(\frac{3\pi}{2}\right)}{\sinh\left(\frac{3\pi}{1}\right)} = -5.38 \cdot 10^{-5} \\ n=5 &\rightarrow \frac{2}{5} \sin\left(\frac{5\pi x}{L}\right) \cdot \frac{\sinh\left(\frac{5\pi y}{L}\right)}{\sinh\left(\frac{5\pi W}{L}\right)} = \frac{2}{5} \sin\left(\frac{5\pi}{2}\right) \cdot \frac{\sinh\left(\frac{5\pi}{2}\right)}{\sinh\left(\frac{5\pi}{1}\right)} = 6.03 \cdot 10^{-8} \\ n=7 &\rightarrow \frac{2}{7} \sin\left(\frac{7\pi x}{L}\right) \cdot \frac{\sinh\left(\frac{7\pi y}{L}\right)}{\sinh\left(\frac{7\pi W}{L}\right)} = \frac{2}{7} \sin\left(\frac{7\pi}{2}\right) \cdot \frac{\sinh\left(\frac{7\pi}{2}\right)}{\sinh\left(\frac{7\pi}{1}\right)} = -8.04 \cdot 10^{-11} \end{aligned}$$

$$T_{m+1,n} + T_{m-1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n} = 0$$

$$\begin{cases} T_2 + T_3 + 700 + 400 - 4T_1 = 0 \\ T_1 + T_m + 500 + 400 - 4T_2 = 0 \\ T_1 + T_m + 100 + 700 - 4T_3 = 0 \end{cases} \Leftrightarrow \begin{cases} -4T_1 + T_2 + T_3 = -1100 \\ T_1 - 4T_2 + 0T_3 = -1246 \\ T_1 + 0T_2 - 4T_3 = -1146 \end{cases}$$

Sous forme matricielle:

$$[A] \cdot [T] = [C]$$

$$\begin{bmatrix} -4 & 1 & 1 \\ 1 & -4 & 0 \\ 1 & 0 & -4 \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} -1100 \\ -1246 \\ -1146 \end{bmatrix}$$

$$[T] = [A]^{-1} \cdot [C], [A]^{-1} = \frac{1}{\det A} \cdot A^{ct}, \det A = (-1)^{i+j} |A|_i, A^t = [M_{ij}] \Rightarrow A^{ct} = [M_{ji}]$$

$$\det A = (-1)^{1+1} \begin{vmatrix} -4 & 0 \\ 0 & -4 \end{vmatrix} + \dots = -56$$

$$A^t = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}, A^{ct} = \begin{bmatrix} M_{11} & M_{21} & M_{31} \\ M_{12} & M_{22} & M_{32} \\ M_{13} & M_{23} & M_{33} \end{bmatrix}$$

$$M_{11} = (-1)^{i+j} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} \begin{vmatrix} -4 & 0 \\ 0 & -4 \end{vmatrix} = 16;$$

$$M_{12} = (-1)^{i+j} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = (-1)^{1+2} \begin{vmatrix} 1 & 0 \\ 1 & -4 \end{vmatrix} = 4;$$

$$\dots = \dots \begin{vmatrix} \dots & \dots \\ \dots & \dots \end{vmatrix} = \dots \begin{vmatrix} \dots & \dots \\ \dots & \dots \end{vmatrix} = \dots;$$

$$A^t = \begin{bmatrix} 16 & 4 & 4 \\ 4 & 15 & 1 \\ 4 & 1 & 15 \end{bmatrix}, A^{ct} = \begin{bmatrix} 16 & 4 & 4 \\ 4 & 15 & 1 \\ 4 & 1 & 15 \end{bmatrix};$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \frac{1}{-56} \begin{bmatrix} 16 & 4 & 4 \\ 4 & 15 & 1 \\ 4 & 1 & 15 \end{bmatrix} \cdot \begin{bmatrix} -1100 \\ -1246 \\ -1146 \end{bmatrix};$$

$$\begin{cases} T_1 = \frac{1}{-56} [16 \cdot (-1100) + 4 \cdot (-1246) + 4 \cdot (-1146)] = 485,14 \text{ C} \\ T_2 = \frac{1}{-56} [4 \cdot (-1100) + 15 \cdot (-1246) + 1 \cdot (-1146)] = 432,78 \text{ C} \\ T_3 = \frac{1}{-56} [4 \cdot (-1100) + 1 \cdot (-1246) + 15 \cdot (-1146)] = 407,78 \text{ C} \end{cases}$$

Vérification

$$-4 \cdot (485,14) + 432,78 + 407,78 = -1100 \text{ C}$$